1)-

[redacted]

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad A = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 & -2 \\ 1 & 5 & 3 \\ 0 & -2 & -5 \end{pmatrix}$$

a)- $A - 2B$

$$\begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} - 2\begin{pmatrix} 3 & 0 & -2 \\ 1 & 5 & 3 \\ 0 & -2 & -5 \end{pmatrix} = \begin{pmatrix} -5 & -1 & 2 \\ -3 & -9 & -5 \\ 1 & 3 & 11 \end{pmatrix}$$

b)- traza(AB)

$$\begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} -5 & -1 & 2 \\ -3 & -9 & -5 \\ 1 & 3 & 11 \end{pmatrix} = \begin{pmatrix} -4 & 2 & -15 \\ 3 & -5 & 4 \\ -1 & 11 & 18 \end{pmatrix}$$

$$tr(AB) = -4 + -5 + 18 = 19$$

2)

a). Gaus Jordan

$$[A|I_3] = \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{F_1 - F_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{F_3 + 2F_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{array}\right) \xrightarrow{F_2 - F_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -2 & 2 & 1 \end{pmatrix}$$

$$A = \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 2 & 0 \end{array}\right) \qquad \begin{aligned} \det A &= 1 + 2 + 0 - (2 + 0 + 0) \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$\therefore$ A es inverdible

b) -

$$(X^t A^t - BA^t)^t = B \quad / \text{prop } t$$

$$(X^t A^t)^t - (BA^t)^t = B$$

$$(X^t)^t (A^t)^t - (BA^t)^t = B$$

$$XA - (BA^t)^t = B$$

$$XA = B + (BA^t)^t \quad /$$

$$X = (B + (BA^t)^t) A^{-1}$$

c) -

$$B \cdot A^t = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 3 & 1 \\ 3 & 2 & 3 \end{pmatrix}$$

$$B + (BA^t)^t = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 3 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 4 \\ 2 & 5 & 3 \\ 2 & 2 & 4 \end{pmatrix}$$

$$(B + (BA^t)^t) A^{-1} = \begin{pmatrix} 2 & 4 & 4 \\ 2 & 5 & 3 \\ 2 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 \\ 6 & -1 & -2 \\ -2 & 4 & 2 \end{pmatrix} = X$$

3) a) usando teorema de propiedades de determinante → triangulo superior.

$$A=\begin{pmatrix}0&1&1&1\\0&2&1&0\\1&1&1&2\\3&1&0&a\end{pmatrix}\xrightarrow[F_1 \text{ por } F_3]{\text{cambio}}\begin{pmatrix}1&1&1&2\\0&2&1&0\\0&1&1&1\\3&1&0&a\end{pmatrix}\xrightarrow{F_4-3F_1}\begin{pmatrix}1&1&1&2\\0&2&1&0\\0&1&1&1\\0&-2&-3&a-6\end{pmatrix}$$

$$\xrightarrow[F_2 \text{ por } F_3]{\text{cambio}}\begin{pmatrix}1&1&1&2\\0&1&1&1\\0&2&1&0\\0&-2&-3&a-6\end{pmatrix}\xrightarrow[F_4+2F_2]{F_3-2F_2}\begin{pmatrix}1&1&1&2\\0&1&1&1\\0&0&-1&-2\\0&0&-1&a-4\end{pmatrix}\xrightarrow{F_4-F_3}\begin{pmatrix}1&1&1&2\\0&1&1&1\\0&0&-1&-2\\0&0&0&a-2\end{pmatrix}$$

$$\text{Det } A = 1\cdot 1\cdot -1\cdot(a-2) = -a+2$$

b) con $a=3$

$\text{Det } A = -3+2 = -1$ $\qquad$ $\text{Det } A \neq 0$

Por lo tanto la matriz A tiene inversa.